JCBI
Journal of Computing & Biomedical Informatics
ISSN: 2710 - 1606
Volume 08 Issue 02
2025

Research Article
https://doi.org/10.56979/802/2025

Decision Making Framework for Phishing Incident Response Using Intuitionistic Fuzzy Trapezoidal Preference Relations

Muhammad Touqeer[1], Syeda Sadia Gilani[1], Adeel Ahmed[2], Awais Mahmood[2], and Muhammad Munwar Iqbal[3*]

[1]Department of Mathematical Sciences, University of Engineering and Technology, Taxila, Pakistan.
[2]Department of Computer Science, Szabist University Islamabad, Pakistan.
[3]Department of Computer Science, University of Engineering and Technology, Taxila, Pakistan.
[*]Corresponding Author: Muhammad Munwar Iqbal. Email: munwariq@gmail.com

Received: Novermber 11, 2024 **Accepted:** March 01, 2025

Abstract: Decision-making often involves uncertainty, requiring precise methodologies to ensure accuracy and reliability. This paper presents an intuitionistic fuzzy trapezoidal preference relation (IFTrPR)-based decision-making framework that integrates multiplicative consistency for improved priority weight assessment. The proposed approach determines intuitionistic fuzzy trapezoidal priority weight vectors and ranks alternatives using the technique for order preference by similarity to the ideal solution (TOPSIS). To enhance the consistency of priority weights, a Linear Decision Model (LDM) is employed, effectively capturing decision-makers' perceptions. Additionally, Model 1 is introduced to compute priority weights based on intuitionistic fuzzy trapezoidal numbers (IFTNs) across various alternatives. The integration of fuzzy logic and optimization techniques strengthens the framework's ability to handle complex decision-making problems. A comparative analysis with hierarchical fuzzy systems (HFS) demonstrates that the proposed method enhances accuracy and reliability in priority weight assessment. Furthermore, the study provides a systematic approach to handling linguistic variables in decision-making, particularly in the representation of membership (MS) and non-membership.

Keywords: Intuitionistic Fuzzy Trapezoidal Preference Relation (IFTrPR); Linear Decision Model (LDM); Multiplicative Consistency; Decision-Making; Priority Weights; Fuzzy Logic.

1. Introduction

In recent decades, many decision making problems emerging under conditions of uncertainty and imprecision have been studied as such problems can be seen to be of relevance in engineering economics and Hence, out of the many theories that have emerged to tackle such problems, theories dealing with the fuzzy set theory[1] even its relative, intuitionistic fuzzy sets (IFS) [2,3,4] say as well as trapezoidal fuzzy sets (TrFS) can be considered to be very efficient. Among all the approaches created to overcome these issues, one of the most effective techniques is FS theory together with its extended versions including IFS [17,18] and TrFS.These problems that include both membership and non-membership degree frameworks are integrated into these structured systems to cope with different degrees of uncertainty, and vagueness prevailing in decision making processes. Intuitionistic fuzzy trapezoidal preference relations (IFTrPR)[14,22] extend trapezoidal fuzzy preference relations (TrFPR) by using positive and negative degrees of the IFS and encompassing all the advantages of separability and flexibility of the trapezoidal membership functions. Used to solve this gree frameworks are implemented into these structured systems in order to accommodate for these different degrees of uncertainty and vagueness present throughout the decision making process. IFTrPR generalized TrFPR using both positive and negative degrees of IFS while at same time apply the advantage of the flexibility of trapezoidal membership functions.aches developed to address these problems, fuzzy set theory and its extensions, such as IFS and TrFS, have emerged as powerful tools. These frameworks suggest how membership as well as non-membership degrees can be introduced into the decision making procedures systematically and hence, imitating a more precise ambiguity and vagueness.

The IFTrPR can be defined as an extension of the TrFPR, which took benefits of the IFS and the flexibility of membership functions in trapezoidal shape. Out of the many approaches evolved to handle these problems, fuzzy set theory and even its relative, viz.IFS as well as TrFS are found to be very effective management. Among the many approaches developed to address these problems, FS theory and its extensions, such as IFS and TrFS, have emerged as powerful tools. Both membership and non-membership degree frameworks are implemented into these structured systems in order to accommodate for these different degrees of uncertainty and vagueness present throughout the decision making process.IFTrPR generalize TrFPR using both positive and negative degrees of IFS while at same time apply the advantage of the flexibility of trapezoidal membership functions. Approaches developed to address these problems, FS theory and its extensions, such as IFS and TrFS, emerged like powerful resource. These frameworks provide a structured way to incorporate both membership, non-membership degrees, thus capturing more nuanced uncertainty and ambiguity in decision-making processes.

IFTPR [14,30] are an extension of TrFPR combining the advantages of IFS with the flexibility of trapezoidal membership functions. The employment of IFTPRs provide a more enriched representation over the preferences of the expert such that it can handle a case where the decision makers give probabilities along with their preferences. The subject may therefore benefit from developing systematic approaches to solving decision-making problems employing IFTPRs to form a research agenda.

Among the stock of approaches to address DM problems, one of the most widely used technique for the Order preference by Similarity to Ideal Solution (TOPSIS). Therefore, in this paper, an attempt is made to applied IFTPRs within the TOPSIS framework to handle MCDM problems in which preference information is in terms of IFTrN[26]. TOPSIS[29] enables an easy determination of the ranking of the options nearer the plus ideal solution than the minus ideal solution.

In addition to this TOPSIS[7,9] based methodology, this research explores other processes on how determine priority weight vectors of decision criteria as IFTPRs. The least deviation model and another model, another model (1), use the least two kinds of strategies. The least deviation model ensures that the forced preference relations are not far from the required preference structure and yet the experts' judgments are as close as possible. On the other hand, Model 1 proposes a new concept of setting the priority weights that make the decision making framework robust.

2. Intuitionistic fuzzy preference relation

Definition 2.1 Intuitionistic fuzzy preference relation: An IFPR [10, 16, 18, 22, 30] P on X is defined as the following IF judgment matrix $L = (l_{pq})_{n\times n} \subset X \times X$, where $L_{pq} = (\chi_{pq}, \psi_{pq})$, and L_{pq} is an IFV. Here, χ_{pq} is the certainty degree to which alternative x_p is preferred to alternative x_q, and ψ_{pq} is the certainty degree to which alternative x_p is not preferred to alternative x_q:

$$0 \le \chi_{pq} + \psi_{pq} \le 1, \quad \chi_{pq} = \psi_{pq}, \quad p, q = 1,2,\ldots,n. \tag{1}$$

2.1. Consistency of Intuitionistic Fuzzy Preference Relation

This section introduces the order consistency of Intuitionistic Fuzzy Preference Relations (IFPR). Furthermore, some other characteristics of multiplicatively consistent IFPRs are also discussed.

Definition 2.2 *An IFPR $L = (l_{pq})_{n\times n}$ with $l_{pq} = (\chi_{pq}, \psi_{pq})$ is said to be an order-consistent IFPR if it satisfies $M_{ps} \ge l_{pt}$ for all $p = 1,2,\ldots,n$.*

Each element (χ, ψ) satisfies the conditions of intuitionistic fuzzy sets, ensuring:

$$\chi + \psi \le 1. \tag{2}$$

According to IFPR L, the following properties hold:

1. $l_{pp} = (0.5,0.5)$ for all $p \in \{1,2,3,4\}$.
2. The membership degree χ satisfies:

$$\chi_{p4} \leq \chi_{p3} \leq \chi_{p2} \leq \chi_{p1}.$$

From this, the ranking indicated by IFPR L is:

$$\boxed{x_1 \succ x_2 \succ x_3 \succ x_4}$$

Definition 2.3 *An IFPR $L = (l_{pq})_{n\times n}$ is said to be multiplicatively consistent if it satisfies the following multiplicative transitivity conditions:*

$$\chi_{pq}\chi_{qr}\chi_{rp} = \chi_{pr}\chi_{rq}\chi_{qp}, \quad \forall p,q,r = 1,2,\dots,n. \tag{3}$$

$$\psi_{pq}\psi_{qr}\psi_{rp} = \psi_{pr}\psi_{rq}\psi_{qp}, \quad \forall p,q,r = 1,2,\dots,n. \tag{4}$$

2.1.1. Operations on IFNs

Let IFNs be (χ_1,ψ_1) and (χ_2,ψ_2). The hesitation degree is $\pi = 1-\chi-\psi$. The operations are defined as follows:

- **Addition:**
$$(\chi_1,\psi_1) + (\chi_2,\psi_2) = (\min(\chi_1+\chi_2,1), \max(\psi_1+\psi_2-1,0)).$$
- **Subtraction:**
$$(\chi_1,\psi_1) - (\chi_2,\psi_2) = (\max(\chi_1-\chi_2,0), \max(\psi_1-\psi_2,0)).$$
- **Multiplication:**
$$(\chi_1,\psi_1)\cdot(\chi_2,\psi_2) = (\chi_1\cdot\chi_2, \psi_1+\psi_2-\psi_1\cdot\psi_2).$$
- **Division:**
$$\frac{(\chi_1,\psi_1)}{(\chi_2,\psi_2)} = \left(\frac{\chi_1}{\chi_2}, \frac{\psi_1}{1-\psi_2}\right), \quad \text{for}\chi_2 > 0, \psi_2 < 1.$$
- **Scalar Multiplication:**
$$k\cdot(\chi_1,\psi_1) = \begin{cases} (\min(k\cdot\chi_1,1), \max(1-k\cdot(1-\psi_1),0)), & k>0 \\ (0,1), & k=0 \\ \text{Undefined}, & k<0 \end{cases}$$
- **Scalar Division:**
$$\frac{(\chi_1,\psi_1)}{k} = (\min(\chi_1/k,1), \max(\psi_1/k,0)), \quad \text{for}k > 0.$$

3. Trapezoidal Fuzzy Preference Relation

Definition 3.1 *A Trapezoidal Fuzzy Preference Relation (TrFPR) generalizes classical preference relations. TrFPR can be defined as follows:*

Let $L_{pq} = (l_{pq})_{n\times n}$ be a preference matrix, where $L_{pq} = (l_{pq1}, l_{pq2}, l_{pq3}, l_{pq4})$ is a Trapezoidal Fuzzy Preference Number (TFPN):

$$\frac{1}{9} \leq l_{pq1} \leq l_{pq2} \leq l_{pq3} \leq l_{pq4} \leq 9, \quad \forall p,q = 1,2,\dots,n. \tag{5}$$

Definition 3.2 *A Trapezoidal Fuzzy Number (TrFN) is characterized by four parameters:*

$$\tilde{A} = (a,b,c,d)$$

where:

- a and d are the lower and upper bounds.
- b and c define the core where membership is 1.

3.1. Membership Function of Trapezoidal Fuzzy Number

The MS function $\chi_{\tilde{A}}(x)$ of TrFN $\tilde{A} = (a,b,c,d)$ is defined as:

$$\chi_{\tilde{A}}(x) = \begin{cases} 0, & x<a \\ \frac{x-a}{b-a}, & a\leq x<b \\ 1, & b\leq x\leq c \\ \frac{d-x}{d-c}, & c<x\leq d \\ 0, & x>d \end{cases}$$

1. **Support**: Fuzzy set $\tilde{A}$ has support equal to the interval $[a,d]$ where in the membership function is greater

than zero.

2. **Core**: The main body of $\tilde{A}$ is the interval $[b, c]$, at which the MS degree is equal to 1.

3. **Interpretation**: The interval $[b, c]$ defines the most possible values for the given FN; the intervals of $[a, b]$ and $[c, d]$ define the diminishing of MS degree down to the support borders.

3.2. Properties of Trapezoidal Numbers

A TrFN is represented as $T = (a, b, c, d)$, where:

- a and d are the lower and upper bounds, respectively.
- b and c are the core bounds, where $a \leq b \leq c \leq d$.

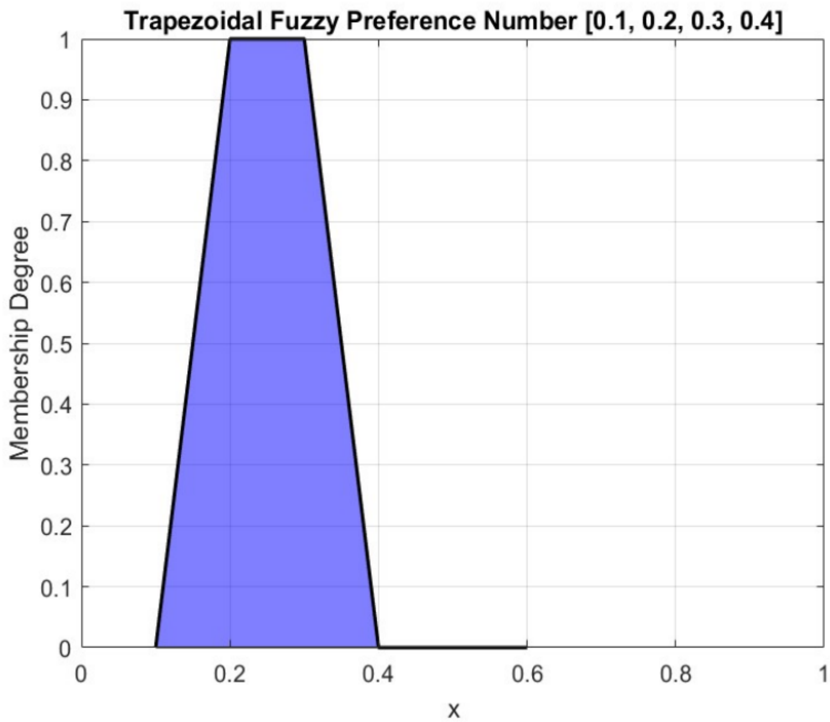

Figure 1. Trapezoidal fuzzy numbers

Given two trapezoidal numbers $T_1 = (a_1, b_1, c_1, d_1)$ and $T_2 = (a_2, b_2, c_2, d_2)$, the operations are defined as follows:

1. Sum

$$T_1 + T_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

2. Difference

$$T_1 - T_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$$

3. Multiplication

$$T_1 \cdot T_2 = (\min(a_1a_2, a_1d_2, d_1a_2, d_1d_2), b_1b_2, c_1c_2, \max(a_1a_2, a_1d_2, d_1a_2, d_1d_2))$$

4. Quotient (Division)

$$T_1 \div T_2 = (\min\left(\frac{a_1}{d_2}, \frac{a_1}{a_2}, \frac{d_1}{d_2}, \frac{d_1}{a_2}\right), \frac{b_1}{b_2}, \frac{c_1}{c_2}, \max\left(\frac{a_1}{d_2}, \frac{a_1}{a_2}, \frac{d_1}{d_2}, \frac{d_1}{a_2}\right))$$

where $a_2, b_2, c_2, d_2 \neq 0$.

5. Scalar Multiplication

For a scalar $k \geq 0$,

$$k \cdot T_1 = (ka_1, kb_1, kc_1, kd_1)$$

If $k < 0$,

$$k \cdot T_1 = (kd_1, kc_1, kb_1, ka_1)$$

6. Power Rule

For a positive integer n,

$$T_1^n = (a_1^n, b_1^n, c_1^n, d_1^n)$$

For a negative integer n,

$$T_1^n = \left(\frac{1}{d_1^n}, \frac{1}{c_1^n}, \frac{1}{b_1^n}, \frac{1}{a_1^n}\right)$$

3.3. Arithmetic Operations on Trapezoidal Fuzzy Numbers

Basic arithmetic operations can be defined on TrFN, which is useful in fuzzy decision-making and preference aggregation.

1. **Addition**: Given $\tilde{A} = (a_1, b_1, c_1, d_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2)$, the addition of two TrFNs is:

$$\tilde{A} + \tilde{B} = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$$

2. **Multiplication (by Scalar)**: If λ is a positive scalar, then for $\tilde{A} = (a, b, c, d)$,

$$\lambda \cdot \tilde{A} = (\lambda a, \lambda b, \lambda c, \lambda d)$$

3. **Approximate Multiplication**: Multiplying two TrFNs can be complex, but approximate methods are sometimes used, particularly in applications like decision-making.

TrFNs are widely applied in scenarios where uncertainty is bounded and the range of values within the "core" is specified. Their trapezoidal shape is easy to work with, making them popular in FS theory and FPRs.

Definition 3.4 *Assume that* $\tilde{A} = (a_{pq})_{n\times n}$ *is a fuzzy square matrix defined over the finite set of alternatives* $X = \{x_1, x_2, \dots, x_n\}$ *and* $\tilde{a}_{pq} = (a_{pq1}, a_{pq2}, a_{pq3}, a_{pq4})$ *denote positive TrFNs If the following conditions satisfied, the matrix* $\tilde{A} = (\tilde{a}_{pq})_{n\times n}$ *is called a multiplicative trapezoidal fuzzy matrix (MTrF):*

$$\begin{cases} a_{pq1} \times a_{pq4} = a_{pq2} \times a_{pq3} = a_{pq3} \times a_{pq2} = a_{pq4} \times a_{pq1} = 1, & p, q = 1,2, \dots, n, p \neq q, \\ \tilde{a}_{pp} = (1,1,1,1), & p = 1,2, \dots, n. \end{cases} \tag{6}$$

where $\tilde{a}_{pq} = (a_{pq1}, a_{pq2}, a_{pq3}, a_{pq4})$, is the positive trapezoidal number denoting fuzzy degree of alternative x_p over x_q $\forall p, q = 1,2, \dots n$

4. Intuitionistic fuzzy trapezoidal preference relation[14,30]

Let $L = (l_{pq})_{n\times n}$ represent a FPR on finite set of alternatives $l = (l_1, l_2, \dots, l_n)$. For any $p, q = 1,2, \dots, n$, l satisfies the following conditions:

$$\begin{aligned} & a_{pq} + d_{qp} = d_{pq} + a_{qp} = b_{pq} + c_{qp} = c_{pq} + b_{qp} = 1, \\ & a_{pq} \leq b_{pq} \leq c_{pq} \leq d_{pq}, \\ & a_{pp} = b_{pp} = c_{pp} = d_{pp} = 0.5, \\ & \chi_{pq}, \psi_{pq} \in [0,1], \quad \chi_{pq} + \psi_{pq} \leq 1, \\ & \chi_{pq} = \chi_{qp}, \quad \psi_{pq} = \psi_{qp}, \quad \chi_{pp} = 1, \quad \psi_{pp} = 0.5. \end{aligned} \tag{7}$$

It is thus possible to define an IFTrPR [14,26,30] as $l_{pq} = ([a_{pq}, b_{pq}, c_{pq}, d_{pq}]; \chi_{pq}, \psi_{pq})$ for all $p, q = 1,2, \dots, n$ where χ_{pq} point at the degree of indeterminacy is denoted as π_{pq} which is also equal to $1 - \chi_{pq} - \psi_{pq}$. The IFTrPR enables generating a refined description of preferences that reflect human judgments, which are typically uncertain and vague.

4.1. Consistency of Intuitionistic fuzzy trapezoidal prference relations

Definition 4.1 *Let* $L = (l_{pq})_{n\times n}$ *be an IFTrPR characterized by* $l_{pq} = ([a_{pq}, b_{pq}, c_{pq}, d_{pq}]; \zeta_{pq}, \eta_{pq})$. *If for any* $p, q = 1,2, \dots, n$ *there exists:*

$$a_{pq} + d_{pq} + a_{kp} + d_{kp} + a_{qk} + d_{qk} \approx 3 \tag{8}$$

$$b_{pq} + c_{pq} + b_{kp} + c_{kp} + b_{qk} + c_{qk} \approx 3 \tag{9}$$

$$\chi_{pq}\chi_{qk}\chi_{kp}\psi_{qp}\psi_{kq}\psi_{pk} = \chi_{qp}\chi_{kq}\chi_{pk}\psi_{pq}\psi_{qk}\psi_{kp} \quad (10)$$

Then $Z = (z_{pq})_{n\times n}$ is called a consistent IFTrFPR.

4.2. Hamming Distance[19,20]

Definition 4.2 *Let* $l_{pq1} = ([a_{pq1}, b_{pq1}, c_{pq1}, d_{pq1}]; \chi_{pq1}, \psi_{pq1})$ *and* $l_{pq2} = ([a_{pq2}, b_{pq2}, c_{pq2}, d_{pq2}]; \chi_{pq2}, \psi_{pq2})$ *be two IFTrPNs. Then normalized Hamming distance between* l_{pq1} *and* l_{pq2} *is defined as:*

$$\begin{aligned} d_{pq} = d(l_{pq1}, l_{pq2}) = \tfrac{1}{8}(&|(1+\chi_{pq1}-\psi_{pq1})a_{pq1} - (1+\chi_{pq2}-\psi_{pq2})a_{pq2}| \\ &+|(1+\chi_{pq1}-\psi_{pq1})b_{pq1} - (1+\chi_{pq2}-\psi_{pq2})b_{pq2}| \\ &+|(1+\chi_{pq1}-\psi_{pq1})c_{pq1} - (1+\chi_{pq2}-\psi_{pq2})c_{pq2}| \\ &+|(1+\chi_{pq1}-\psi_{pq1})d_{pq1} - (1+\chi_{pq2}-\psi_{pq2})d_{pq2}|) \end{aligned} \quad (11)$$

For Ideal solution

$$\begin{aligned} d^+_{pq} = d(l^+_{pq}, l^+_q) = \tfrac{1}{8}(&|(1+\chi_{pq}-\psi_{pq1})z^1_{pq} - (1+\chi_q-\psi_q)z^+_q| \\ &+|(1+\chi_{pq}-\psi_{pq})z^2_{pq} - (1+\chi_q-\psi_q)z^+_q| \\ &+|(1+\chi_{pq}-\psi_{pq})z^3_{pq} - (1+\chi_q-\psi_q)z^+_q| \\ &+|(1+\chi_{pq}-\psi_{pq})z^4_{pq} - (1+\chi_q-\psi_q)z^+_q|) \end{aligned} \quad (12)$$

this For Anti-Ideal solution

$$\begin{aligned} d^-_{pq} = d(\tilde{l}_{pq}, \tilde{l}^-_q) = \tfrac{1}{8}(&|(1+\chi_{pq}-\psi_{pq})z^1_{pq} - (1-\chi^-_q-\psi^-_q)z^{1-}_q| \\ &+|(1+\chi_{pq}-\psi_{pq})z^2_{pq} - (1-\chi^-_q-\psi^-_q)z^{2-}_q| \\ &+|(1+\chi_{pq}-\psi_{pq})z^3_{pq} - (1-\chi^-_q-\psi^-_q)z^{3-}_q| \\ &+|(1+\chi_{pq}-\psi_{pq})z^4_{pq} - (1-\chi^-_q-\psi^-_q)z^{4-}_q|) \end{aligned} \quad (13)$$

4.3. A priority weight derivation procedure with intuitionistic fuzzy trapezoidal preference relation

In new real world decision making contexts, the priority weight derivation method ITrFPR as introduced in [14] enables a DM to determine an IFTrPR that may not be fully consistent. To this end, the difference matrix $D = (d_{pq})_{n\times n}$, derived from the deviation between the judgement matrix $L = (l_{pq})_{n\times n}$ and the consistent IFTrPR $\tilde{L} = (\tilde{l}_{pq})_{n\times n}$, is used to measure inconsistency. These matrices' elements are presented as $l_{pq} = ([a_{pq}, b_{pq}, c_{pq}, d_{pq}]; \zeta_{pq}, \eta_{pq})$.

A priority weight derivation [14] procedure ITrFPR in a real decision-making process allows a DM to provide an IFTrPR that may not satisfy complete consistency. In such cases, the deviation between given judgment matrix $L = (l_{pq})_{n\times n}$ and the consistent IFTrPR $\tilde{L} = (\tilde{l}_{pq})_{n\times n}$ is used to represent the difference. Elements in $L = (l_{pq})_{n\times n}$ and $\tilde{L} = (\tilde{l}_{pq})_{n\times n}$ are expressed as $l_{pq} = ([a_{pq}, b_{pq}, c_{pq}, d_{pq}]; \chi_{pq}, \psi_{pq})$.

$$\begin{aligned} \tilde{l}_{pq} &= \left([\tilde{a}_{pq}, \tilde{b}_{pq}, \tilde{c}_{pq}, \tilde{d}_{pq}]; \tilde{\chi}_{pq}, \tilde{\psi}_{pq}\right) \\ &= \left(\left[\tfrac{\gamma}{2}(w^a_p - w^d_q) + 0.5,\ \tfrac{\gamma}{2}(w^b_p - w^c_q) + 0.5,\ \tfrac{\gamma}{2}(w^c_p - w^b_q) + 0.5,\ \tfrac{\gamma}{2}(w^d_p - w^a_q) + 0.5\right],\ \tilde{\chi}_{pq}, \Psi_{pq}\right)). \end{aligned}$$

Where:

$$\tilde{\chi}_{pq} = \frac{2w^\chi_p}{(w^\chi_p - w^\psi_p) + \lambda(w^\chi_q - w^\psi_q) + \lambda + 1},$$

$$\tilde{\psi}_{pq} = \frac{2\lambda w^\chi_q}{(w^\chi_p - w^\psi_p) + \lambda(w^\chi_q - w^\psi_q) + \lambda + 1}.$$

The deviation is shown as:

$$
\begin{aligned}
d_{pq}^{a} &= \frac{\gamma}{2}(w_p^a - w_q^d) + 0.5 - a_{pq}, \quad p,q = 1,2,\dots,n;\; p \neq q,\\
d_{pq}^{b} &= \frac{\gamma}{2}(w_p^b - w_q^c) + 0.5 - b_{pq}, \quad p,q = 1,2,\dots,n;\; p \neq q,\\
d_{pq}^{c} &= \frac{\gamma}{2}(w_p^c - w_q^b) + 0.5 - c_{pq}, \quad p,q = 1,2,\dots,n;\; p \neq q,\\
d_{pq}^{d} &= \frac{\gamma}{2}(w_p^d - w_q^a) + 0.5 - d_{pq}, \quad p,q = 1,2,\dots,n;\; p \neq q.
\end{aligned}
$$

$$
\tilde{\alpha}_{pq} = \frac{2w_p^{\chi}}{(w_p^{\chi} - w_p^{\psi}) + \lambda(w_q^{\chi} - w_q^{\psi}) + \lambda + 1} - \chi_{pq},
$$

$$
\tilde{\beta}_{pq} = \frac{2\lambda w_q^{\chi}}{(w_p^{\chi} - w_p^{\psi}) + \lambda(w_q^{\chi} - w_q^{\psi}) + \lambda + 1} - \psi_{pq}.
$$

The smaller the absolute deviation, the higher the consistency of IFTrPR. Thus, a fractional programming model is constructed as follows:

$$
\text{Model1:} \quad \min Z = \sum_{i=1}^{n} \sum_{l=k+1}^{n} \left(d_{kl}^{g+} + d_{kl}^{g-} + d_{kl}^{h+} + d_{kl}^{h-} + d_{kl}^{i+} + d_{kl}^{i-} + d_{kl}^{j+} + d_{kl}^{j-} + \alpha_{kl}^{+} + \alpha_{kl}^{-} + \beta_{kl}^{+} + \beta_{kl}^{-} \right) \tag{14}
$$

Subject to

$$
\begin{cases}
\frac{\gamma}{2}(w_k^g - w_l^j) + 0.5 - g_{kl} - d_{kl}^{g+} + d_{kl}^{g-} = 0, & k,l = 1,\dots,n;\; k \neq l,\\
\frac{\gamma}{2}(w_k^h - w_l^i) + 0.5 - h_{kl} - d_{kl}^{h+} + d_{kl}^{h-} = 0, & k,l = 1,\dots,n;\; k \neq l,\\
\frac{\gamma}{2}(w_k^i - w_l^h) + 0.5 - i_{kl} - d_{kl}^{i+} + d_{kl}^{i-} = 0, & k,l = 1,\dots,n;\; k \neq l,\\
\frac{\gamma}{2}(w_k^j - w_l^g) + 0.5 - j_{kl} - d_{kl}^{j+} + d_{kl}^{j-} = 0, & k,l = 1,\dots,n;\; k \neq l,\\
\frac{2w_k^{\chi}}{(w_k^{\chi} - w_k^{\psi}) + \lambda(w_l^{\chi} - w_l^{\psi}) + \lambda + 1} - \chi_{kl} - \alpha_{kl}^{+} + \alpha_{kl}^{-} = 0, & \\
\frac{2\lambda w_l^{\chi}}{(w_k^{\chi} - w_k^{\psi}) + \lambda(w_l^{\chi} - w_l^{\psi}) + \lambda + 1} - \psi_{kl} - \beta_{kl}^{+} + \beta_{kl}^{-} = 0, & \\
0 \le w_k^g \le w_k^h \le w_k^i \le w_k^j \le 1, \quad 0 \le \sum_{k=1}^{n} w_k^g \le 1 \le \sum_{k=1}^{n} w_k^j, & k,l = 1,\dots,n;\; k \neq l,\\
w_k^{\chi}, w_k^{\psi} \in [0,1], \quad w_k^{\chi} + w_k^{\psi} \le 1, & k,l = 1,\dots,n;\; k \neq l,\\
\sum_{\substack{l=1\\ l\neq k}}^{n} w_l^{\chi} \le w_k^{\psi}, \quad w_k^{\chi} + n - 2 \ge \sum_{\substack{l=1\\ l\neq k}}^{n} w_l^{\psi}, & k,l = 1,\dots,n;\; k \neq l,\\
d_{kl}^{g+}, d_{kl}^{g-}, d_{kl}^{h+}, d_{kl}^{h-}, d_{kl}^{i+}, d_{kl}^{i-}, d_{kl}^{j+}, d_{kl}^{j-} \le 0, & \\
d_{kl}^{g+} d_{kl}^{g-} = 0, \quad d_{kl}^{h+} d_{kl}^{h-} = 0, \quad d_{kl}^{i+} d_{kl}^{i-} = 0, \quad d_{kl}^{j+} d_{kl}^{j-} = 0, & k,l = 1,\dots,n;\; k \neq l,\\
\alpha_{kl}^{+}, \alpha_{kl}^{-}, \beta_{kl}^{+}, \beta_{kl}^{-} \le 0, \quad \alpha_{kl}^{+}\alpha_{kl}^{-} = 0, \quad \beta_{kl}^{+}\beta_{kl}^{-} = 0, & k,l = 1,\dots,n;\; k \neq l.
\end{cases}
$$

By solving this model, the intuitionistic fuzzy trapezoidal weight $w = (w_1, w_2, \dots, w_n)$ can be derived, where each $w_p = ([w_p^a, w_p^b, w_p^c, w_p^d]; w_p^{\chi}, w_p^{\psi})$. If the optimal objective value Z equals 0, the original $L = (l_{pq})_{n\times n}$ is consistent.

4.4. TOPSIS Method: Technique for order preference by similarity to the Ideal Solution

TOPSIS [9,29] is a multi-criteria Decision Making (DM) technique used to assist the decision makers for ranking alternatives from a set of solutions depending on several attributes and for assigning which of the available options is 'nearest' to the ideal solution. It is most advantageous where one comes across conflicting criterion stems from the decision, since it returns the totally rational value of compromise distance in the optimization of the solution space when measured using the geometric mean of the distance from the ideal solution solution and the anti-ideal solution. TOPSIS is considered as one of the best methods of multi criteria

Decision Making. This measure has been proposed by Hwang and Yoon in 1981[11] and family of measures assumes that the smallest distance to the PIS and the greatest distance to the NIS should identify the best compromise solution. The PIS illustrates an example of a solution that provides benefits that meets definite maximum criteria while the NIS illustrates the same solution that provides costs that meet definite minimum criteria. The evaluation and ranking of the decision-making alternatives based on the relative proximity of the positive ideal solution in decision-making process makes TOPSIS a simple, efficient and practical tool in multitude of applications.

TOPSIS method is powerful in the field of practical applications of the multiple criteria decision making. Some notable applications include: The areas of application comprise the supply chain management and storeroom management of a hospital, equipment and treatment, production and construction engineering, environmental assessment, and financial and economic assessment. It might be important to notice that TOPSIS is applied widely and provides sufficiently good solutions for decision making in many fields. It is easy to understand, reasoned and easy to compute, that's why it is quite popular among practitioners as well as researchers.

TOPSIS method in its essence deals with the identification of an alternative which has minimum geometric distance from the ideal solution and maximum distance from anti-ideal solution. This approach can be applied practically in most areas of practice including but not limited to supply chain management, health care, engineering and among others.

Below is the step-by-step process of the TOPSIS method:

Step 1: Define the Decision Matrix

The decision matrix reflects quantitative performance of m compromises ($A_1, A_2, \dots, A_m$) with reference to n attributes ($C_1, C_2, \dots, C_n$). The decision matrix is structured as follows:

$$D = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix}$$

where x_{ij} represents the performance of alternative A_i under criterion C_j.

Step 2: Normalize the Decision Matrix

To eliminate the influence of different units, the decision matrix is normalized using the following formula:

$$r_{pq} = \frac{x_{pq} - \min(x_{pq})}{\max(x_{pq}) - \min(x_{pq})} \tag{15}$$

where r_{pq} is the normalized value of x_{pq}. This step ensures that all criteria values are dimensionless and comparable.

Step 3: Calculate the Weighted Normalized Decision Matrix

The weighted normalized decision matrix is obtained by multiplying each normalized value by the corresponding weight of the criterion:

$$v_{pq} = w_q \cdot r_{pq} \tag{16}$$

where v_{pq} is the weighted normalized value, w_q is the weight of criterion q, and r_{pq} is the normalized value.

Step 4: Determine the Ideal and Anti-Ideal Solutions

The ideal solution (A^+) represents the best values for each criterion, and the anti-ideal solution (A^-) represents the worst values:

$$A^+ = [v_1^+, v_2^+, \dots, v_n^+]; max(\chi^+), min(\psi^+) \tag{17}$$

$$A^- = [v_1^-, v_2^-, \dots, v_n^-]; min(\chi_-), max(\psi^-) \tag{18}$$

where: $v_j^+ = \max(v_{pq})$ for benefit criteria, and $v_q^- = \min(v_{pq})$ for cost criteria.

Step 5: Compute the Distance to Ideal and Anti-Ideal Solutions

The distance of each alternative from the ideal (D_i^+) and anti-ideal (D_i^-) solutions are calculated as follows:

$$D_p^+ = \sqrt{\sum_{q=1}^{n} (v_{pq} - v_q^+)^2}, \quad (19)$$

$$D_p^- = \sqrt{\sum_{q=1}^{n} (v_{pq} - v_q^-)^2} \quad (20)$$

Step 6: Calculate the Relative Closeness to Ideal Solution

The relative closeness of each alternative to ideal solution is given by:

$$R_p = \frac{D_p^-}{D_p^+ + D_q^-} \quad (21)$$

where R_p is relative closeness of alternative A_p to ideal solution. A higher value R_p indicates greater closeness to the ideal solution.

Step 7: Rank the Alternatives

Finally, the alternatives are ranked based on their relative closeness values (R_p). The alternative with the highest R_p value is considered the best choice.

5. An approach to decision making based on Intuitionistic fuzzy trapezoidal preference relation

To find the ranking of IFTrPR by using a technique for order preference by similarity to the ideal solution (TOPSIS)

5.1. Algorithim

- 1:Develop a decision matrix based on IFTrPR .
- 2:Determine the priority-weighted vector.
- 3:Computed the weighted normalized decision matrix.
- 4:For,it is necessary to find what is optimal and what is optimal to avoid.
- 5:Determine the distance to ideal and anti-ideal solutions
- 6:Choose number for the measure of relative closeness.
- 7:Rank the alternatives.

5.2. Medical Scenario Example

A public health department in a city is required to mitigate an outbreak of a communicable illness. This outbreak is threatening to human life, increases a burden to health centers, and requires immediate action and prevention. To address this problem, this department must examine several elements to develop an efficient countermeasure plan. The decision-making process is based on three main components, represented by $(Y_p = (y_1, y_2, y_3)$:

- y_1 = contagion control measures (e.g, vaccination, quarantine),
- y_2 = Healthcare physical assets, Health facility, Health products and equipment.
- y_3 = community awareness and support programs have been eliminated.

Step(1): Construct a decision matrix according to IFTrPR matrix.

$$X = \left[\begin{pmatrix} ([0.5,0.5,0.5,0.5];1,0) \\ ([0.7,0.8,0.9,1.0];0.7,0.3 \\ ([0.2,0.5,0.6,0.9];0.9,0.0 \end{pmatrix} \begin{pmatrix} ([0.0,0.1,0.2,0.3];0.3,0.7 \\ ([0.5,0.5,0.5,0.5];1,0) \\ ([0.1,0.3,0.5,0.6]:0.1,0.2 \end{pmatrix} \begin{pmatrix} ([0.1,0.4,0.5,0.8];0.9,0.0) \\ ([0.4,0.5,0.7,0.9];0.2,0.1) \\ ([0.5,0.5,0.5,0.5];1,0) \end{pmatrix} \right]$$

The graphical representation of intuitionistic fuzzy trapezoidal preference numbers are shown in fig (1)and fig (2).

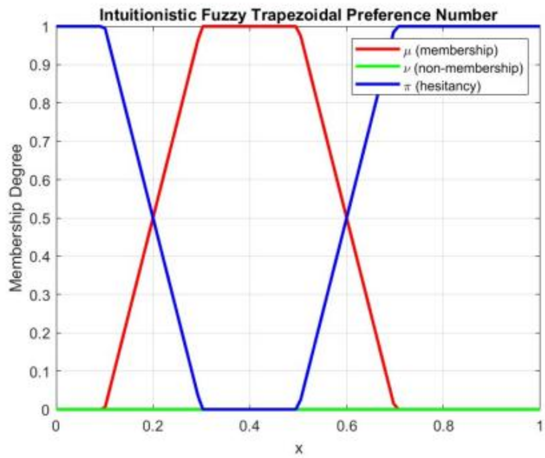

Figure 2. An Intuitionistic fuzzy trapezoidal numbers

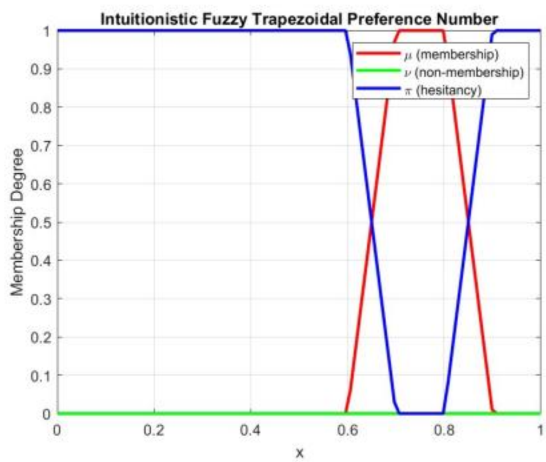

Figure 3. An Intuitionistic fuzzy trapezoidal numbers

Step(2): To find the priority weight vector by using Model(1)
Then we get weights $W_1 = ([0.13,0.24,0.40,0.15]; 0.29,0.70)$

$W_2 = ([0.86,0.86,0.94,0.97]; 0.64,0.29)$

$W_3 = ([0.00,0.48,0.53,1.00]; 0.00,0.93)$

Step(3): Calculate weight decision matrix
calculate weight decision matrix by using equation(12) then we get

$$\begin{bmatrix} \begin{pmatrix} ([0.65,0.12,0.2,0.075]; 1,0) \\ ([0.60,0.67,0.85,0.9]; 0.45,0.50) \\ ([0.00,024,0.32,0.9]; 0.0,0.99) \end{pmatrix} \\ \begin{pmatrix} ([0.00,0.02,0.08,0.04]; 0.19,0.91) \\ ([0.43,0.43,0.47,0.48]; 1,0) \\ ([0.00,0.14,0.26,0.6]; 0.0,0.94) \end{pmatrix} \\ \begin{pmatrix} ([0.01,0.09,0.2,0.12]; 0.26,0) \\ ([0.34,0.34,0.67,0.87]; 0.13,0.36) \\ ([0.0,0.24,0.265,0.5]; 0,0) \end{pmatrix} \end{bmatrix}$$

Step(4): Determine the ideal and anti- ideal solutions we analyze the ideal and anti ideal solutions by using eq(13) and eq(14). then we get A^{+}=([0.60,0.67,0.85,0.9]; 0.45,0),([0.43,0.43,0.47,0.6]; 0.64,0), ([0.34,0.43,0.67,0.87]; 0.26,0)

A^{-}=([0.00,0.12,0.2,0.075]; 0.00,0.99),([0.00,0.02,0.08,0.04]; 0.0,0.94), ([0.0,0.93,0.2,0.12]; 0.0,0.36).

Step(5): Compute distance to ideal and anti ideal solution

Now analyze distance to ideal and anti ideal solution from equation (11) and equation (12) then we get the distance

$$d_{pq}^{+} = \begin{bmatrix} 0.3514 & 0.39075 & 0.2976 \\ 0.18875 & 0.0246 & 0.1415 \\ 0.5456 & 0.38815 & 0.2388 \end{bmatrix}$$

and

$$d_{pq}^{-} = \begin{bmatrix} 0.1815 & 0.0038 & 0.0645 \\ 0.3573 & 0.37 & 0.1884 \\ 0.00049 & 0.0064 & 0.1149 \end{bmatrix}$$

$$D_p^{+} = \sum_{j=1}^{n} d_{ij}^{+}$$

$$D_p^{-} = \sum_{j=1}^{n} d_{ij}^{-}$$

$D_1^{+} = 1.0397,\ D_2^{+} = 0.35485,\ D_3^{+} = 1.17255.$

$D_1^{-} = 0.2498,\ D_2^{-} = 0.9157,\ D_3^{-} = 0.12179.$

Step(6): Calculate the relative closeness

Now we compute the relative closeness to the ideal solution by using equation (18). we find it $CC_1 = 0.8064,\ CC_2 = 0.2973,\ CC_3 = 0.9059.$

Step(7) Rank the alternative

We explore the ranking from relative closeness Y_3 is in the highest rank $Y_3 > Y_1 > Y_2$.

6. Comparative analysis

A comparison was also conducted to determine how effective the proposed method of finding priority weight vectors was used in other research studies. In particular, the TOPSIS method and least deviation model are chosen as basic methods of comparison. In this study, the Technical advantage ranking Technique based on TOPSIS was used since it was observed to be quite strong in ranking decision alternatives based on the closeness to ideal solution being sought. In this case, the least deviation model which is non – optimization model was used to arrive at a priority weight vector that gives minimum deviations which in turn allow for sustaining and achieving the accuracy of the weight assignment process. The comparison made in the analysis of results obtained from the proposed technique with those two approaches exerted benefits in knowing the reliability and practicality of the proposed method, which provided clues on its ability to solve decision making problems under uncertainties. An approach to decision making based on TrPR priority weight vectors with other methods used in the literature. Particularly, the TOPSIS method and the least deviation model were selected as benchmark techniques. In this study, TOPSIS, a methodology that has been proven effective in ranking decision alternatives in terms of their closeness to the ideal solution was applied due to its strength in decision making. In contrast, the least deviation model, a non-optimization based method was solved to find the priority weight vector which gives minimum deviations and thus helps in sustaining and achieving accuracy of the weight assignment process. The analysis made in comparing the results of proposed method with these two approaches proved to be beneficial in determining it's reliability and practicability, which gave indication of its effectiveness in handling decision making problems under uncertainties.

An approach to decision making based on trapezoidal preference relation

Now find the ranking of TrFPR by using TOPSIS method

- 1:Construct a decision matrix according to TFPR matrix.
- 2:Calculate the priority weight vectors

- 3:Calculate the weighted normalized decision matrix.
- 4:Determine the ideal and anti-ideal solution.
- 5:Calculate the distance to ideal and anti ideal solutions
- 6:Calculate the relative closeness.
- 7:Rank the alternatives.

Example

Step(1): To construct the matrix according to condition of TrFPR Matrix that is

$$\begin{bmatrix} [0.5,0.5,0.5,0.5] & [0.3,0.4,0.6,0.7] & [0.2,0.3,0.4,0.5] \\ [0.7,0.6,0.4,0.3] & [0.5,0.5,0.5,0.5] & [0.4,0.5,0.8,0.9] \\ [0.5,0.4,0.3,0.2] & [0.9,0.8,0.5,0.4] & [0.5,0.5,0.5,0.5] \end{bmatrix}$$

The graphical representation is shown in fig(4)

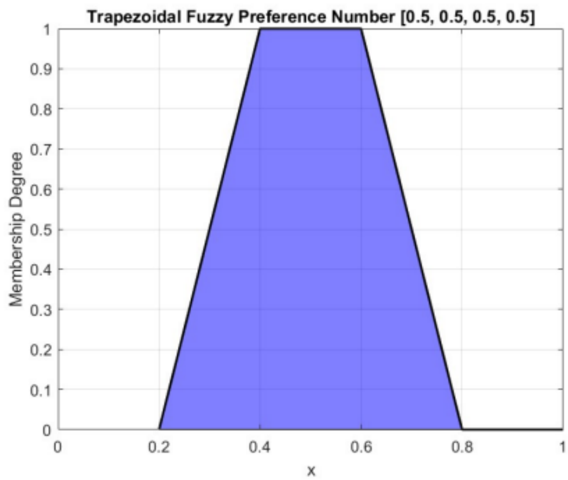

Figure 4. Trapezoidal fuzzy preference numbers

Step(2): To calculate weight vectors

we compute the priority weight vectors by using least deviation model then we get

$$W_1 = [0.40,0.62,0.62,0.32],$$

$W_2 = [0.40,0.99,0.99,0.56],$

$w_3 = [0.15,,0.15,0.34,0.31].$

Step(3): Calculated the weighted decision matrix

We get the weighted normalized decision matrix by using eq(14). we get

$$v_{pq} = \begin{bmatrix} [0.2,0.31,0.31,0.16] & [0.12,0.25,0.37,0.22] & [0.08,0.19,0.25,0.16] \\ [0.28,0.59,0.39,0.17] & [0.20,0.50,0.50,0.28] & [0.16,0.50,0.80,0.50] \\ [0.08,0.06,0.10,0.06] & [0.13,0.4,0.17,0.12] & [0.075,0.075,0.17,0.155] \end{bmatrix}$$

Step(4): Determine the ideal and anti-ideal solutions

we analyze the ideal and anti-ideal solutions by using equations (15) and (16) $A^+ = ([028,0.59,0.39,0.17]), ([0.20,0.50,0.50,0.28]), ([0.16,0.50,0.80,0.50])$

and $A^- = ([0.08,0.06,0.10,0.06]), ([0.12,0.25,0.17,0.12]), ([0.07,0.07,0.17,0.15])$

Step(5): Calculate the distance to ideal and anti-ideal solutions

We find the distance to ideal and anti-ideal solutions by utilizing the equations (17) and (18) $D_1^+ = [0.47,0.66,0.40,0.12]$ $D_2^+ = [0.36,0.36,0.36,0.36]$ $D_3^+ = [0.44,0.80,0.67,0.23]$

and

$D_1^- = [0.16,0.26,0.33,0.16]$ $D_2^- = [0.23,0.68,0.84,0.45]$ $D_3^- = [0.08,0.25,0.13,0.08]$

Step(6): Calculate the relative closeness

We calculate the relative closeness to ideal solutions by applying equation (18)

$$CC_1 = [1.68,0.90,0.43,0.19]$$

$$CC_2 = [0.44,0.3,0.35,0.61]$$
$$CC_3 = [1.42,1,0.64,0.44]$$

Step(7): Rank the alternatives
We get the ranking through calculating relative closeness that is $Y_3 > Y_1 > Y_2$

7. Conclusion

The work here used Model (1) to deduce the priority weight vectors of IFTrPR, and the integration of the TOPSIS ranking method has provided an overall decision-making framework. The comparison was made by using the same TOPSIS ranking procedure on the TrFPR. Interestingly, the rankings were considerably similar between the intuitionistic fuzzy trapezoidal method and the trapezoidal fuzzy method. It is this consistency which confirms the effectiveness and applicability of the suggested Model (1) and the TOPSIS-based ranking method for solving DM problems with fuzzy and intuitionistic fuzzy contexts. The implications of these findings stress the contingency of the discussed approach in low-vagrant problem instances that call for fancy and accurate ranking under the uncertainty.

References

1. K. Atanassov, “Intuitionistic fuzzy sets,” Fuzzy Sets and Systems, vol. 20, no. 1, pp. 87-96, 1986.
2. K. T. Atanassov, “More on intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 33, pp. 37-46, 1989.
3. K. T. Atanassov, G. Gargov, “Interval-valued intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 31, no. 3, pp. 343-349, 1989.
4. K. T. Atanassov and G. Gargov, “Uncertainty and fuzziness in interval-valued intuitionistic fuzzy sets,” Fuzzy Sets and Systems, vol. 35, no. 1, pp. 47-58, 1989.
5. H. Bustince, P. Burillo, “Correlation of interval-valued intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 74, pp. 237–244, 1995.
6. H. Bustince, P. Burillo, “Vague sets are intuitionistic fuzzy sets," Fuzzy Sets and Systems, vol. 79, pp. 403-405, 1996.
7. S. M. Chen, L. W. Lee, “Fuzzy multiple attributes group decision-making based on the interval type-2 TOPSIS method," Expert Systems with Applications, vol. 37, no. 4, pp. 2790-2798, 2010.
8. S. J. Chen, J. M. Tan, “Handling multicriteria fuzzy decision-making problems based on vague sets," Fuzzy Sets and Systems, vol. 67, no. 2, pp. 163-172, 1994.
9. Chakraborty, S., Yeh, C. H. (2009). A simulation comparison of normalization procedures for TOPSIS. Mathematical and Computer Modelling, 50(7-8), 861-867. A discussion of normalization techniques used in the TOPSIS method.
10. H. Gao, C. Ma, and J. Yang, “A method for intuitionistic fuzzy preference relations and its application in multi-criteria decision making,” Journal of Computational and Applied Mathematics, vol. 236, no. 8, pp. 1881-1895, 2012.
11. Hwang, C. L., Yoon, K. (1981). Multiple attribute decision making methods and applications.
12. D. F. Li, "Multiattribute decision making models and methods using intuitionistic fuzzy sets," Journal of Computer and System Sciences, vol. 70, no. 1, pp. 73-85, 2005.
13. C. T. Lin and C. G. Lee, "Neural fuzzy systems: A neuro-fuzzy synergism to intelligent systems." Prentice-Hall, Inc, 1996.
14. Nie, R.-X., Wang, J.-Q., and Wang, T.-L. “A Three-cycle Decision-making Selection Mechanism with Intuitionistic Trapezoidal Fuzzy Preference Relations." 1 Jan. 2019: 5409 – 5422.
15. E. Szmidt, J. Kacprzyk, “Remarks on some applications of intuitionistic fuzzy sets in decision making," Notes on IFS, vol. 2, pp. 22–31, 1996.
16. E. Szmidt, J. Kacprzyk, “Group decision making under intuitionistic fuzzy preference relations," in: Proceedings of the 7th IPMU Conference, Paris, 1998, pp. 172–178.
17. E. Szmidt, J. Kacprzyk, “Using intuitionistic fuzzy sets in group decision making," Control and Cybernetics, vol. 31, pp. 1037–1053, 2002.
18. E. Szmidt, J. Kacprzyk, “A concept of similarity for intuitionistic fuzzy sets and its use in group decision making," in: Proceedings of the International Joint Conference on Neural Networks and IEEE International Conference on Fuzzy Systems, Budapest, Hungary, 2004, pp. 25-29.
19. V. Torra, “The weighted OWA operator using intuitionistic fuzzy sets and Grey Relational Analysis," Knowledge-Based Systems, vol. 24, no. 1, pp. 19-25, 2011.
20. J. Q. Wang, H. Y. Zhang, and X. M. Liu, “Grey relational analysis method for intuitionistic fuzzy multiple attribute decision making," Expert Systems with Applications, vol. 37, no. 5, pp. 3849-3855, 2010.
21. W. Q. Wang, X. L. Xin, “Distance measure between intuitionistic fuzzy sets," Pattern Recognition Letters, vol. 26, pp. 2063–2069, 2005.
22. R. Wang and Y. L. Li, “A novel approach for group decision-making from intuitionistic fuzzy preference relations and intuitionistic multiplicative preference relations," Information, vol. 9, no. 3, pp. 55, 2018.
23. Z. S. Xu, “Intuitionistic fuzzy aggregation operators," IEEE Transactions on Fuzzy Systems, vol. 15, no. 6, pp. 1179-1187, 2007.
24. Z. Xu, R. R. Yager, “Intuitionistic and Interval-Valued Intuitionistic Fuzzy Preference Relations," IEEE Transactions on Fuzzy Systems, vol. 14, no. 2, pp. 289-300, 2006.
25. Z. Xu, “Intuitionistic preference relations and their application in group decision making," Information Sciences, vol. 177, no. 11, pp. 2363-2379, 2007.
26. J. Yuan, C. Li, “A new method to multi-attributes decision making with intuitionistic trapezoidal fuzzy random

variable," International Journal of Fuzzy Systems, vol. 19, no. 1, pp. 15-26, 2017.
27. T.-M. Yeh, H. Deng, Y.-H. Chang, “Fuzzy Multi-Criteria Analysis for Performance Evaluation of Public Transport Systems," International Journal of Systems Science, vol. 31, no. 4, pp. 391-402, 2000.
28. L. A. Zadeh, “Fuzzy sets," Information and Control, vol. 8, pp. 338-353, 1965.
29. Zavadskas, E. K., Turskis, Z., Kildiene, S. (2015). State of art surveys of overviews on MCDM/MADM methods. Technological and Economic Development of Economy, 21(1), 165-179. A review of multiple-criteria decision-making methods, including TOPSIS.
30. Z. Zhang, W. Zhou, and L. Liu, “Intuitionistic trapezoidal fuzzy numbers and their operations," Computers Mathematics with Applications, vol. 62, no. 3, pp. 1053-1070, 2011.
31. Akram, S., Khalid, H., Rasulov, T., Khalid, M., & Rehman, M. U. (2023). A Nobel Variation of 16th-Order Iterative Scheme for Models in Blood Stream, Chemical Reactor, and Its Dynamics. Lobachevskii Journal of Mathematics, 44(12), 5116-5131.
32. Akram, S., Khalid, M., & Junjua, M. (2023). uD; Altaf, S.; Kumar, S. Extension of King’s Iterative Scheme by Means of Memory for Nonlinear Equations. Symmetry 2023, 15, 1116.