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Identifying Best-Fit Probability Distribution for Modelling Annual Maximum Daily Rainfall

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Abstract: The study examines the annual maximum daily rainfall (AMDR) in Multan, Pakistan, using data from 1951 to 2016. L-Moments (LM), Maximum Likelihood Estimation (MLE), Maximum Product of Spacings (MPS), and Bayesian estimation were the estimation techniques used to estimate the parameters of Pearson Type-III, Gumbel, Weibull, and Generalized Extreme Value (GEV) probability distributions. The models were evaluated using different accuracy measures. Based on the comparative results using accuracy metrics, the Weibull distribution appears to best fit the rainfall data for Multan. Among the parameter estimation methods for the Weibull distribution, MPS performs better than other methods. The identification of the Weibull distribution as best-fit probability model makes it more appropriate for modelling rainfall data and hence planning for water resources, agriculture, and disaster risk reduction in Multan and infrastructure design and flood risk management.

Keywords: Climate change; Estimation methods; Frequency analysis; GEV; Gumbel; Pearson-type-III; Probability distributions; Rainfall

1. Introduction

Climate change has increased the frequency and intensity of extreme weather events worldwide. Each year, millions of people are affected by two of the most devastating natural disasters: floods and extreme rainfalls. Floods and extreme rainfall severely affect human health by causing injuries, drowning, and the spread of waterborne diseases such as diarrhea, cholera, and typhoid [24]. According to a 2018 study by the World Meteorological Organization, floods are responsible for nearly 90% of worldwide disasters that are associated with extreme weather events. These events damage infrastructure, disrupt agriculture, and pose serious threats to public safety and the economy globally. Moreover, agricultural practices can further contribute to environmental degradation, as it has been observed that about 2% of sprayed pesticides vaporize into the atmosphere and may contribute to acid rain formation [26]. Due to South Asia's highly variable rainfall patterns, droughts and floods are common occurrences [1,2]. Pakistan is particularly vulnerable because its water and agricultural resources depend on seasonal rainfall. Multan, a significant agricultural hub, is located in southern Punjab's semi-arid terrain, where rainfall is sparse and largely concentrated during the monsoon season. Since extremely low rainfall can cause water shortages and unexpected excessive rainfalls can cause crop damage and urban flooding, accurate rainfall forecasting is essential for the region.

An approach that is frequently used to study excessive rainfall is rainfall frequency analysis (RFA). Its reliability depends on the use of an appropriate blend of probability distribution and estimation methods [3,4]. Weibull, Gumbel, Pearson Type-III and Generalized Extreme Value (GEV) distributions are commonly employed in hydrology to model rainfall extremes [5]. Bayesian methodologies, Maximum Likelihood Estimation (MLE), Maximum Product of Spacings (MPS), and L-moments are the most commonly used techniques for estimating their parameters [6,7]. Comparisons among different models (combinations of probability distribution and estimation method) are typically done by evaluating model performance using several common performance metrics.

Although rainfall modelling has been done in Pakistan and various other South Asian regions, the modelling of rainfall patterns in the southern part of Punjab, Pakistan, is relatively limited. Most of the previous studies have focused on northern Pakistan or broader regional aspects [8–10]. This study fills that gap by applying four probability distributions and a variety of estimation approaches to Multan's long-term rainfall data. Using some common accuracy indicators, the study evaluated each model's performance in order to identify the most suitable model for rainfall prediction in Multan. The findings of the current study provide important information for the region's agricultural, water resource and disaster risk reduction plans.

2. Study Sites

In Pakistan's southern Punjab region, Multan is a prominent semi-arid city with scorching summers, mild winters, and monsoon-dominated rainfall. Multan District spans 3,721 km^2 and is the most populous district, with 4.7 million inhabitants, as well as the largest urban center in southern Punjab [25]. This city is the focus of the present study. The accuracy and consistency of the annual maximum daily rainfall data from 1951 to 2016, obtained from the Pakistan Meteorological Department, were checked before rainfall frequency analysis. There were no missing data for any year within the range of years considered for the analysis. The yearly maximum of daily rainfall was used as the representative value for each corresponding year. Basic information about the study area and data is given in Table 1.

Table 1. Some basic information about gauging station of cities.

Station No.	City	Time Period	Latitude	Longitude	Elevation(m)
PMD-428	Multan	1951-2016	30.20°N	71.47°E	122

3. Methodology

This study used annual maximum daily rainfall (AMDR) data from 1951 to 2016 in Multan. The data series were modelled using four probability distributions Pearson Type-III, Gumbel, Weibull and Generalized extreme values (GEV). For parameter estimation, L-Moments (LM), Maximum Likelihood (ML), Maximum Product Spacing (MPS), and Bayesian methods were used.

3.1. Candidate Probability distributions

Four distributions: Gamma (Pearson Type-III), Gumbel, Weibull and Generalized Extreme Value (GEV) were investigated in this study. These distributions are commonly applied and found plausible for conducting at-site and regional flood or rainfall frequency analysis. Their probability density and quantile functions are presented in Table 2.

Table 2. Probability density and quantile functions of selected distributions

Dist.	Probability Density Function (pdf)	Quantile Function (QF)
GUM	$f(x)=\frac{1}{\beta}\exp\left\{\frac{-(x-\mu)}{\beta}-\exp\left(\frac{-(x-\mu)}{\beta}\right)\right\}$	$Q(p)=\mu-\beta*\log(1-\log(p))$
GEV	$f(x)=\frac{1}{\beta}\left\{1+k\left(\frac{(x-\mu)}{\beta}\right)\right\}^{-\left(1-\frac{1}{k}\right)}\exp\left\{-\left(1+k\left(\frac{(x-\mu)}{\beta}\right)\right)\right\}^{-\left(\frac{1}{k}\right)}$	$Q(p)=\mu+\frac{\beta}{k}\left\{(-\log(p))^{-k}-1\right\}$
PE-3	$f(x)=\frac{1}{\Gamma k\beta^{k}}\left\{(x-\mu)^{k-1}*\exp\left(\frac{-(x-\mu)}{\beta}\right)\right\}$	$Q(p)=\mu+\beta\left\{G^{-1}(p;k)\right\}$ where G^{-1} is the quantile of gamma distribution

WEIBULL	$f(x)=\frac{\beta}{k}\left\{\left(\frac{x}{k}\right)^{\beta-1}\exp\left(\frac{x}{k}\right)^{\beta}\right\}$	$Q(p)=k\left\{-\log(1-p)\right\}^{\frac{1}{\beta}}$

where the location, scale, and shape parameters are denoted by μ, β, and k, respectively.

3.2. Estimation Methods

The parameters were estimated using L-Moments (LM), Maximum Likelihood Estimation (MLE), Maximum Product of Spacing (MPS), and Bayesian approaches. LM is stable for skewed data, Bayesian estimation considers previous information, MLE works well for large samples, and MPS is reliable when MLE fails. These considerations provide a strong basis for comparisons of distributions when using these estimation strategies. For brevity and simplicity, the mathematical details of these estimation methods are not provided in the text. These details are well documented in many other studies [6,11–17,23].

3.2.1. Method of L-Moments

L-moments, analogous to conventional moments [27], are defined for any random variable x when its mean exists [18]. These L-moments are generally computed through their relationship with the probability weighted moments (PWMs). The PWM of kth order (γ_k) is expressed as:

$$\gamma_k = \int_0^1 x(F)F(x)^k dF, \;\; k = 0, 1, 2,3 \ldots$$

Then the first four L-moments can be computed in terms of PWM by using the following relationships,

$$\lambda_1 = \gamma_0,$$
$$\lambda_2 = 2\gamma_1 - \gamma_0$$
$$\lambda_3 = 6\gamma_2 - 6\gamma_1 + \gamma_0$$
$$\lambda_4 = 20\gamma_3 - 30\gamma_2 + 12\gamma_1 - \gamma_0$$

For any sample data, the first four unbiased estimates of PWM (b_r) can be computed as,

$$b_0 = n^{-1}\sum_{i=1}^{n} x_{i:n}$$
$$b_1 = n^{-1}\sum_{i=2}^{n} \frac{(i-1)}{(n-1)} x_{i:n}$$
$$b_2 = n^{-1}\sum_{i=3}^{n} \frac{(i-1)(i-2)}{(n-1)(n-2)} x_{i:n}$$
$$b_3 = n^{-1}\sum_{i=4}^{n} \frac{(i-1)(i-2)(i-3)}{(n-1)(n-2)(n-3)} x_{i:n}$$

where $x_{i:n}$ is the observed data series arranged in an ascending order. Consequently, the sample L-moments and L-moment ratios can be computed using above relationships defined by Hosking [18] and Hosking and Wallis [19]. Following Hosking and Wallis [19] parameter estimates using the LM method for probability distributions were obtained.

3.2.2. Method of Maximum Likelihood

The Maximum Likelihood (ML) is a widely applied estimation method. It gives those values as parameter estimates for which the likelihood or log-likelihood attains its maximum. Let we have a random sample of n observations $x_1, x_2, x_3 \ldots x_n$ from probability density function $f(x_i/\theta)$ where θ is vector of unknown parameters. Let the likelihood and log-likelihood of n independent observations are $L(\theta) = \prod_{i=1}^{n} f(x_i/\theta)$ and $l(\theta) = \sum_i^n \log f(x_i/\theta)$, respectively. The ML estimate of θ is the value for which $l(\theta)$ attains its maximum.

The maximization is achieved by equating the derivative of $l(\theta)$ with respect to θ to zero. If this process does not yield a closed form solution, then maximization can be achieved numerically through an optimization technique. For the current work, we have applied algorithms available in R-language [20] for numerical optimization.

3.2.3. Method of Maximum Product Spacing

The method of maximum product spacing is based on maximizing the following objective function.

$$MPS(\theta) = \left(\prod_{i=1}^{n+1} D_i\right)^{n+1}$$

or equivalently,

$$MPS(\theta) = \left(\frac{1}{n+1}\right)\sum_{i}^{n+1} \log\left(D_i\right)$$

where $D_i = \begin{cases} \hat{F}(x_1) & for\ i = 1 \\ \hat{F}(x_i) - \hat{F}(x_{i-1}) & for\ i = 2,3,\dots,n \\ 1 - \hat{F}(x_n) & for\ i = n+1 \end{cases}$

Further details of MPS estimation method are provided in [21].

3.2.4. Method of Bayesian Estimation

The Bayesian approach serves as an alternative to the frequentist approach. It assumes the unknown parameters as random variables [23]. One of the core advantages of the Bayesian approach is its ability to merge sample information and prior knowledge about parameters, enabling the estimation of the probability density function of these parameters. The posterior distribution of parameter vector θ, given the data x, is calculated using the Bayes' theorem as,

$$p(\theta/x) = \frac{l\left(\frac{x}{\theta}\right)P(\theta)}{p(x)} \propto l(x/\theta)P(\theta)$$

Where $p(\theta/\text{x})$ is a posterior probability distribution of θ, $l(x/\theta)$ represents the likelihood of data series x given θ, $p(x)$ is referred to as the normalizing constant of the posterior distribution, represents the factor that ensures the total probability of the posterior distribution sums to one, and $P(\theta)$ represents a prior probability distribution for θ.

The sampling from $p(\theta/\text{x})$ can be performed using the Markov Chain Monte Carlo (MCMC) technique. In this study, the sampling from posterior probability distributions is carried out using the Metropolis-Hastings (MH) algorithm, which is an MCMC method. The MH is recognized as a highly efficient algorithm in a variety of applications in different domains [14,15]. The details about the general steps involved in MH algorithm to simulate samples from posterior distribution of parameters can be seen in Xu et al. [17]. The MH algorithm generates a set of parameter vectors θ with tens of thousands of values with density$p(\theta/\text{x})$. These samples can be used to approximate the posterior distribution, find moments (expectations), estimate quantiles, compare models, and compute some difficult integrals. The Bayesian estimate of the unknown parameters under the squared error risk is the mean of the posterior distribution i.e.

$$\hat{\theta} = E(\theta/x) = \int \theta\ p\left(\frac{\theta}{\text{x}}\right)d\theta$$

In general, Monte Carlo simulations are used to estimate$E(\theta/x)$. The MCMC samples from $p(\theta/x)$ are used to approximate the parameter estimates as,

$$\hat{\theta}^{(j)} = E(\theta^{(j)}/x) = \frac{1}{N}\sum_{i}^{N}\theta_i^{(j)} \qquad j = 1,2,3 \dots.$$

3.3 Performance Metrics

To evaluate the performance of competing models, four accuracy measures are also computed. These are; Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Square Error (MSE), and Root Mean Square Percentage Error (RMSPE).

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}\left|F_n(x_i) - \hat{F}(x_i)\right|$$

$$\text{MSE} = \frac{1}{\text{n}}\sum_{i=1}^{n}\left(F_n(x_i) - \hat{F}(x_i)\right)^2$$

$$\text{RMSE} = \sqrt{\frac{1}{\text{n}}\sum_{i=1}^{n}\left(F_n(x_i) - \hat{F}(x_i)\right)^2}$$

$$\text{RMSPE} = \sqrt{\frac{1}{\text{n}}\sum_{i=1}^{n}\left(\frac{F_n(x_i) - \hat{F}(x_i)}{F_n(x_i)}\right)^2} * 100$$

In all above measures of performance, $F(x_i)$ shows the sample/observed cumulative distribution function (CDF) computed using mean rank method as:

$$F_n(x_i) = \frac{i}{n+1},$$

while $\hat{F}(x_i)$ is the expected/theoretical CDF of given probability distribution with parameter estimates from particular estimation method. These performance metrics measure the accuracy of fit and smaller values indicate a better fit. Therefore, while ranking the models (combination of probability distribution and estimation method), a lower value of MAE, MSE, RMSE and RMSPE get a higher rank. To have a single performance indicator, individual ranks are summed to get total rank of any given model.

3.4 Quantile Estimation

The best-fit distribution was chosen and rainfall quantiles for return periods were calculated using

$$x_t = F^{-1}\left(1 - \frac{1}{T}\right)$$

where F^{-1} is the inverse CDF of the best-suited model and T is return period in years taken as 2, 5, 10, 25, 50, 100 and 200.

4. Results and Discussion

We used four estimation techniques and four probability distributions (Pearson Type-III, Gumbel, Weibull, and GEV) to evaluate the Multan's Annual Maximum Daily Rainfall (AMDR) dataset (1951–2016). Therefore, we have 16 competing models (combinations of probability distributions and estimation methods). Quantile estimates, accuracy metrics, GOF results, estimated parameters, and descriptive data for certain return periods are shown in this section.

4.1. Summary of AMDR series

The 66-year Multan AMDR series was examined for descriptive statistics. The coefficient of variation, mean, median, standard deviation, skewness, kurtosis and CV are shown in Table 4.1.

Table 4. Summary statistics of AMDR (Multan, 1951-2016)

City	n	Mean (mm)	Median (mm)	SD	Skewness	Kurtosis	CV
Multan	66	87.33	77.95	47.21	1.16	0.95	0.54

The variability of Multan's AMDR series is moderate (CV = 0.54). While kurtosis (0.95) implies fairly heavy-tailed behavior, positive skewness (1.16) indicates extreme rainfall episodes.

4.2. Parameter Estimation

L-Moments (LM), Maximum Likelihood Estimation (MLE), Maximum Product of Spacings (MPS), and Bayesian techniques were used to estimate the parameters of the four candidate distributions: Pearson Type-III, Weibull, Gumbel, and GEV. The fitted parameters for each approach are displayed in Table 5.

Table 5. Estimated parameters of candidate distributions for Multan

Distribution	Method	Location (μ)	Scale (β)	Shape (k)
PE-III	LM	20.568	40.123	1.234
	MLE	20.123	40.567	1.245
	MPS	20.789	39.876	1.228
	Bayes	20.66	40.34	1.24
Weibull	LM	---	82.346	1.876
	MLE	---	83.112	1.865
	MPS	---	81.987	1.892
	Bayes	---	82.15	1.885
Gumbel	LM	60.789	35.679	---
	MLE	61.234	36.123	---
	MPS	60.456	35.234	---
	Bayes	60.91	36.58	---
GEV	LM	50.123	45.678	–0.123
	MLE	50.678	46.123	–0.115
	MPS	49.876	45.234	–0.128
	Bayes	50.35	45.45	–0.12

All methods' parameter estimates are consistent. GEV exhibits a confined upper tail for the negative shape parameter (k). The consistent performance of LM, MLE, MPS, and Bayesian techniques validates accurate model fitting.

Four accuracy measures were used to assess the model's performance (RMSE, MAE, MSE, RMSPE). A summary of the findings is provided in Table 6.

Table 6. Accuracy metrics for Multan AMDR

Distribution	Method	MSE (rank)	MAE (rank)	RMSE (rank)	RMSPE (rank)	Total RANKS
GEV	LM	0.1343 (3)	0.301 (3)	0.3665 (3)	390.53 (12)	21
	MLE	0.1357 (2)	0.3022 (2)	0.3683 (2)	392.11 (10)	16
	MPS	0.1321 (8)	0.2986 (10)	0.3634 (8)	394.42 (9)	35
	Bayes	0.133 (4)	0.3 (4)	0.364 (5)	391.67 (11)	24
Gumbel	LM	0.1306 (13)	0.2977 (13)	0.3614 (13)	371.99 (13)	52
	MLE	0.1363 (1)	0.3036 (1)	0.3692 (1)	369.69 (16)	19
	MPS	0.1328 (5)	0.2998 (5)	0.3644 (4)	369.85 (15)	29
	Bayes	0.132 (9)	0.299 (8)	0.363 (10)	370.25 (14)	41
Weibull	LM	0.1318 (10)	0.2989 (9)	0.3631 (9)	405.15 (1)	29
	MLE	0.13 (14)	0.2974 (14)	0.3606 (14)	401.54 (4)	46
	MPS	**0.1269** (16)	**0.2942** (16)	**0.3563** (16)	**396.58** (7)	**55**
	Bayes	0.131 (11.5)	0.298 (11.5)	0.362 (11.5)	400.73 (5)	39.5
Pearson-III	LM	0.1323 (6.5)	0.2993 (6.5)	0.3638 (6.5)	401.81 (3)	22.5
	MLE	0.1323 (6.5)	0.2993 (6.5)	0.3638 (6.5)	403.63 (2)	21.5
	MPS	0.1287 (15)	0.2959 (15)	0.3588 (15)	395.82 (8)	53
	Bayes	0.131 (11.5)	0.298 (11.5)	0.362 (11.5)	399.5 (6)	40.5

From the results in Table 6, it is clear that Weibull distribution estimated through maximum product spacing (MPS) turns out to be the most plausible model for modelling rainfall patterns in Multan as it has the highest total rank. Pearson type-III estimated with MPS and Gumbel distribution estimated with L-moments are second and third best choices after Weibull distribution.

4.3. Quantile Estimation

Rainfall quantiles calculated using the best-fit distributions for selected return periods (2, 5, 10, 25, 50, and 100 years) are provided in Table 7.

Table 7. Multan's estimated quantiles (mm)

Dist.	Quant.	Non-exceedance probability and return periods (years)						
		0.5	0.8	0.9	0.96	0.98	0.99	0.995
		2	5	10	25	50	100	200
GEV	Lower Limit	70	112	145	180	210	240	270
	Estimate	76	117	148	191	227	265	308
	Upper Limit	84	125	155	200	235	275	320
	$\hat{\sigma}_q$	3.571	3.316	2.551	5.102	6.378	8.929	12.755
Gumbel	Lower Limit	66	108	135	160	180	198	215
	Estimate	79	117	142	174	198	221	245
	Upper Limit	90	130	150	180	205	229	255
	$\hat{\sigma}_q$	6.122	5.612	3.827	5.102	6.378	7.908	10.204
Weibull	Lower Limit	68	110	140	170	187	200	215
	Estimate	82	126	150	178	196	213	228
	Upper Limit	97	142	165	190	210	225	240
	$\hat{\sigma}_q$	7.398	8.163	6.378	5.102	5.867	6.378	6.378
PE3	Lower Limit	65	105	135	166	189	210	229

Estimate	80	121	147	178	200	222	243
Upper Limit	95	138	160	190	215	236	258
$\hat{\sigma}_q$	7.653	8.418	6.378	6.122	6.633	6.633	7.398

From Table 7, it is clear that generally uncertainty (measured through standard error: $\hat{\sigma}_q$) is large for long return periods. From these results, we see that Multan's AMDR is positively skewed and appropriate for extreme-value distributions, according to this analysis, which follows LM, MLE, MPS, and Bayesian estimation with PE-III, Weibull, Gumbel, and GEV models. The two that consistently outperform the others in terms of accuracy are Weibull and Pearson type-III. While the Gumbel distribution offers simplicity for medium return times, these results are consistent with past regional research [8,22].

Finally, Table 8 shows the goodness of fit of different models assessed through Cramer's von Mises (CcM) test.

Table 8. Goodness-of-fit statistics for candidate distributions

Distribution	Method	CvM
PE-III	LM	0.067
	MLE	0.067
	MPS	0.067
	Bayes	0.005
Weibull	LM	0.079
	MLE	0.079
	MPS	0.079
	Bayes	0.005
Gumbel	LM	0.076
	MLE	0.076
	MPS	0.076
	Bayes	0.005
GEV	LM	0.046
	MLE	0.046
	MPS	0.046
	Bayes	0.005

5. Conclusions

The annual maximum daily rainfall (AMDR) in Multan was analyzed using data from 1951 to 2016. Four estimation techniques (LM, MLE, MPS, and Bayesian), four probability distributions (GEV, Gumbel, PE-III, and Weibull) were used to find the most plausible model for modelling rainfall patterns. The performance of different models was compared through several common performance metrics.

The results indicated that the Weibull distribution estimated through maximum product spacing outperformed the other competing models. It is evident from the results that, after the Weibull distribution estimated using MPS, Pearson type-III (estimated with MPS) and Gumbel (estimated with LM) are second and third best choices, respectively.

These results offer helpful directions for engineers and planners creating drainage systems, flood control structures, and water resource plans in Multan. Safety against infrequent extreme events will be improved by using predictions for different return periods. Further, future research may incorporate seasonal or non-stationary analysis to account for the effects of climate change.

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